10th Class 2021

Math (Science)	Group-I	PAPER-II
Time: 2.10 Hours	(Subjective Type)	Max. Marks: 60

(Part-I)

2. Write short answers to any SIX (6) questions: (12)

(i) Define exponential equation.

Ans In an equation, if variable occurs in exponent, then it is called exponential equation.

(ii) Solve by factorization: $x^2 - 11x = 152$

Ans

$$x^2 - 11x = 152$$
$$x^2 - 11x - 152 = 0$$
$$x^2 - 19x + 8x - 152 = 0$$
$$x(x - 19) + 8(x - 19) = 0$$
$$(x - 19)(x + 8) = 0$$
$$x - 19 = 0 \quad \text{or} \quad x + 8 = 0$$

for $x - 19 = 0$

$\Rightarrow \quad x = 19$

for $x + 8 = 0$

$\Rightarrow \quad x = -8$

$\therefore \quad S.S = \{19, -8\}$

(iii) Solve: $x^2 + 2x - 2 = 0$

Ans Here, $a = 1$, $b = 2$, $c = -2$

We may solve the above equation through quadratic formula, so

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-2 \pm \sqrt{(2)^2 - 4(1)(-2)}}{2(1)} = \frac{-2 \pm \sqrt{4 + 8}}{2}$$
$$= \frac{-2 \pm \sqrt{12}}{2} = \frac{-2 \pm 2\sqrt{3}}{2} = -1 \pm \sqrt{3}$$

(iv) Evaluate: $(1 - 3\omega - 3\omega^2)^5$

Ans Given:

$$(1 - 3w - 3w^2)^5$$

By taking common, we get

$$= [1 - 3(w + w^2)]^5$$

As we know that: $\quad w + w^2 = -1$

$$= [1 - 3(-1)]^5$$
$$= (1 + 3)^5$$
$$= 4^5 = 1024$$

(v) Find the product of complex cube roots of unity.

Ans Three cube roots of unity are:

$$1, \frac{-1+\sqrt{-3}}{2} \text{ and } \frac{-1-\sqrt{-3}}{2}$$

The product of cube roots of unity $= (1)\left(\frac{-1+\sqrt{3}}{2}\right)\left(\frac{-1-\sqrt{-3}}{2}\right)$

$$= (-1)^2 - (\sqrt{-3})^2$$
$$= \frac{1-(-3)}{4}$$
$$= \frac{1+3}{4} = \frac{4}{4} = 1$$

i.e., $(1)(w)(w^2) = 1$ or $w^3 = 1$

(vi) If the ratios $3x + 1 : 6 + 4x$ and $2 : 5$ are equal, find the value of x.

Ans $(3x + 1) : (6 + 4x) = 2 : 5$

Product of means = Product of extremes

$$(6 + 4x) \times 2 = (3x + 1) \times 5$$
$$12 + 8x = 15x + 5$$
$$8x - 15x = 5 - 12$$
$$-7x = -7$$
$$x = \frac{-7}{-7}$$
$$x = 1$$

(vii) If $y \, \alpha \, \frac{1}{x}$ and $y = 4$ when $x = 3$, find x when $y = 24$.

Ans Given that $y \, \alpha \, \frac{1}{x}$

$$\Rightarrow \quad y = \frac{k}{x} \qquad \text{(i)}$$

Put y = 4 and x = 3 in (i)

$$4 = \frac{k}{3}$$

$$\Rightarrow \quad k = 12$$

Now we have to find x when y = 24.

Put y = 24 and k = 12 in (i)

$$24 = \frac{12}{x}$$

$$24x = 12$$

$$x = \frac{12}{24}$$

$$x = \frac{1}{2}$$

(viii) **Find ω^2, if $\omega = \frac{-1+\sqrt{-3}}{2}$.**

Ans

$$\omega = \frac{-1+\sqrt{-3}}{2}$$

Squaring $(\omega)^2 = \left(\frac{-1+\sqrt{-3}}{2}\right)^2$

$$= \frac{(-1)^2 + (\sqrt{-3})^2 + 2(-1)(\sqrt{-3})}{2^2}$$

$$= \frac{1+(-3)-2\sqrt{-3}}{4} = \frac{1-3-2\sqrt{-3}}{4}$$

$$= \frac{-2-2\sqrt{-3}}{4} = \frac{(-1-\sqrt{-3})}{2}$$

$$\omega^2 = \frac{-1-\sqrt{-3}}{2}$$

So, if $\omega = \frac{-1+\sqrt{-3}}{2}$

$$\omega^2 = \frac{-1-\sqrt{-3}}{2}$$

(ix) Find a third proportional to: $(x-y)^2, x^3-y^3$.

Ans Let c be the third proportional, then

$$(x-y)^2 : (x^3-y^3) :: (x^3-y^3) : c$$

be in proportion.

We know, product of extreme = product of mean

$$\Rightarrow \quad c(x-y)^2 = (x^3-y^3)\times(x^3-y^3)$$

$$\Rightarrow \quad c = \frac{\cancel{(x-y)}(x^2+xy+y^2)\cancel{(x-y)}(x^2+xy+y^2)}{\cancel{(x-y)^2}}$$

$$c = (x^2+xy+y^2)(x^2+xy+y^2)$$

$$= (x^2+xy+y^2)^2$$

3. Write short answers to any SIX (6) questions: (12)

(i) Resolve into partial fractions: $\dfrac{x-11}{(x-4)(x+3)}$

Ans $$\frac{x-11}{(x-4)(x+3)} = \frac{A}{x-4} + \frac{B}{x+3}$$

$$x-11 = A(x+3) + B(x-4) \qquad \text{(i)}$$

Put $x = 4$, $x = -3$ in (i)

Firstly, $4-11 = A(4+3) + B(4-4)$

$$-7 = A(7) + 0$$

$$\Rightarrow \quad 7A = -7$$

$$\boxed{A = -1}$$

And $-3-11 = A(-3+3) + B(-3-4)$

$$-14 = 0 + B(-7)$$

$$\Rightarrow \quad -7B = -14$$

$$\boxed{B = 2}$$

So, $$\frac{x-11}{(x-4)(x+3)} = \frac{-1}{x-4} + \frac{2}{x+3}$$

(ii) What are partial fractions?

Ans Partial fractions can be define as:

Decomposition of resultant fraction $\dfrac{N(x)}{D(x)}$, with $D(x) \neq 0$, when.

(a) $D(x)$ consists of non-repeated linear factors.

(b) $D(x)$ consists of repeated linear factors.
(c) $D(x)$ consists of non-repeated linear factors.
(d) $D(x)$ consists of repeated irreducible quadratic factors.

(iii) If $X = \{1, 4, 7, 9\}$ and $Y = \{2, 4, 5, 9\}$, then find $Y \cap X$.

Ans Given, $X = \{1, 4, 7, 9\}$, $Y = \{2, 4, 5, 9\}$

$$Y \cap X = \{2, 4, 5, 9\} \cap \{1, 4, 7, 9\}$$
$$= \{4, 9\}$$

(iv) Define an Onto function.

Ans A function $f : A \to B$ is called an onto function, if every element of set B is an image of at least one element of set A *i.e.*, Range of $f = B$.

For example, if $A = \{0, 1, 2, 3\}$ and $B = \{1, 2, 3\}$, then $f : A \to B$ such that $f = \{(0, 1), (1, 2), (2, 3), (3, 2)\}$. Here Range $f = \{1, 2, 3\} = B$.

Thus f so defined is an onto function.

(v) If $A = N$ and $B = W$, then find the value of $B - A$.

Ans

$$A = N = \{1, 2, 3, ...\}$$
$$B = W = \{0, 1, 2, 3, ...\}$$
$$B - A = \{0, 1, 2, 3, ...\} - \{1, 2, 3, ...\}$$
$$= \{0\}$$

(vi) If $L = \{a, b, c\}$, $M = \{d, e, f, g\}$, then find two binary relations in $L \times M$.

Ans $L \times M = \{a, b, c\} \times \{d, e, f, g\}$

$$L \times M = \begin{Bmatrix} (a, d), (a, e), (a, f), (a, g) \\ (b, d), (b, e), (b, f), (b, g) \\ (c, d), (c, e), (c, f), (c, g) \end{Bmatrix}$$

$$R_1 = \{(a, d), (b, f), (c, g)\}$$
$$R_2 = \{(a, e), (b, d), (c, e)\}$$

(vii) Find the arithmetic mean by direct method for the set of data: 200, 225, 350, 375, 270, 320, 290.

Ans The arithmetic Mean:

$$\bar{X} = \frac{\Sigma X}{n}$$
$$= \frac{200 + 225 + 350 + 375 + 270 + 320 + 290}{7}$$

$$= \frac{2030}{7}$$

$$\boxed{\bar{X} = 290}$$

(viii) Define class mark.

Ans For a given class, the average of that class obtained by dividing the sum of upper and lower class limits by 2, is called the midpoint or **class mark** of that class.

(ix) Name two measures of central tendency.

Ans Following are the two measures of central tendency:

1. Arithmatic Mean   2. Median

4. Write short answers to any SIX (6) questions: (12)

(i) Find 'r', when $l = 56$ cm and $\theta = 45^o$.

Ans $l = 56$ cm, $\theta = 45^o$, r = ?

By converting the θ into radians,

$$45^o = 45 \times \frac{\pi}{180} \text{ radian}$$

$$= \frac{\pi}{4} \text{ radians}$$

We have,

$$l = r\theta$$

$$\Rightarrow \quad r = \frac{l}{\theta}$$

$$= \frac{56}{\frac{\pi}{4}} = \frac{56 \times 4}{\pi}$$

$$r = 71.27 \text{ cm}$$

(ii) Define radian measure of an angle.

Ans The angle subtended at the centre of the circle by an arc, whose length is equal to the radius of the circle is called one Radian.

(iii) Express the angle 315^o into radian.

Ans We know

$$180^\circ = \pi \text{ rad.}$$

$$1^\circ = \frac{\pi}{180} \text{ rad.}$$

$$315^\circ = 315 \times \frac{\pi}{180} \text{ rad.}$$

$$= \frac{315}{180} \pi \text{ rad.}$$

$$= \frac{7}{4} \pi \text{ radians}$$

(iv) State theorem of componendo and dividendo.

Ans If $a : b = c : d$, then

(i) $a + b : a - b = c + d : c - d$

and (ii) $a - b : a + b = c - d : c + d$

(v) Find the fourth proportional to 8, 7, 6.

Ans Let the fourth proportional is x:

$$8 : 7 :: 6 : x$$

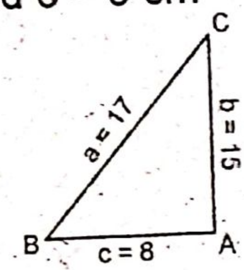
$$8 \times x = 7 \times 6$$

$$x = \frac{42}{8}$$

$$x = \frac{21}{4}$$

(vi) In a ΔABC, $a = 17$ cm, $b = 15$ cm and $c = 8$ cm, find $m\angle B$.

Ans for $a = 17$ cm, $b = 15$ cm and $c = 8$ cm



Pythagora's theorem

$$a^2 = b^2 + c^2$$

$$(17)^2 = (15)^2 + (8)^2$$

$$289 = 225 + 64$$

$$289 = 289$$

$\therefore$ ABC is a right angled triangle.

So, $\tan\angle B = \dfrac{\text{Opp. side}}{\text{Adj. side}}$

$$\tan B = \frac{15}{8}$$

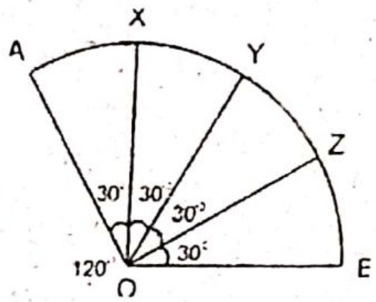
$$B = \tan^{-1}\frac{15}{8} = 61.9^\circ$$

(vii) Divide an arc of any length into four equal parts.

Ans **Steps of construction:**

(i) Divide an arc AB. The central angle of arc is 120°.

(ii) Divide 120° central angle into four equal parts each of size 30°.



(iii) Produce these angles met AB at point A, X, Y, Z and B.

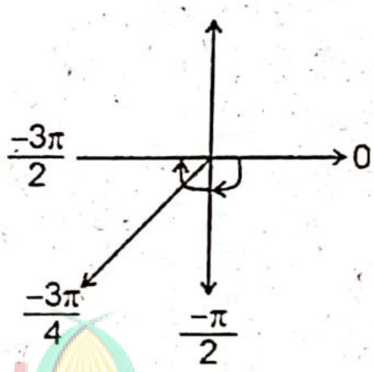
(iv) Arc AB has been divided into four equal parts.

(viii) Write the closest quadrantal angles between which the angle $\frac{-3\pi}{4}$ lies.

Ans

Therefore, the closest quadrantal angles between the angle $-3\frac{\pi}{4}$ lies are:

$$-\frac{\pi}{2} \quad \text{and} \quad -3 = \frac{\pi}{2}$$



(ix) Verify: $(1 \sin\theta)(1+\sin\theta) = \cos^2\theta$

Ans L.H.S

$$(1-\sin\theta)(1+\sin\theta)$$

Using $(a-b)(a+b)$

$$= (1)^2 - \sin^2\theta$$
$$= 1 - \sin^2\theta$$

Using identity

$$\sin^2\theta + \cos^2\theta = 1$$
$$\cos^2\theta = 1 - \sin^2$$
$$\Rightarrow \quad 1 - \sin^2\theta$$
$$= \cos^2\theta = \text{R.H.S}$$
$$\therefore \quad (1-\sin\theta)(1+\sin\theta) = \cos^2\theta$$

(Part-II)

NOTE: Attempt THREE (3) questions in all. But question No. 9 is Compulsory.

Q.5.(a) Solve by factorization: $\frac{x+1}{x}+\frac{x}{x+1}=\frac{25}{12}$. **(4)**

Ans $$\frac{x+1}{x}+\frac{x}{x+1}=\frac{25}{12}$$

Multiply both sides by 12x (x + 1)

$$\frac{x+1}{x}\times 12x(x+1)+\frac{x}{(x+1)}\times 12x(x+1)=\frac{25}{12}.12(x+1)$$

$$12(x+1)(x+1)+12x^2=25\times(x+1)$$

$$12(x^2+2x+1)+12x^2=25(x^2+x)$$

$$12x^2+24x+12+12x^2=25x^2+25x$$

$$24x^2+24x+12=25x^2+25x$$

Take all terms to right side,

$$25x^2+25x-24x^2-24x-12=0$$

$$x^2+x-12=0$$

$$x^2+4x-3x-12=0$$

$$x(x+4)-3(x+4)=0$$

$$(x+4)(x-3)=0$$

Either $x+4=0$ or $x-3=0$

If $x+4=0$

$\Rightarrow$ $x=-4$

or $x-3=0$

$\Rightarrow$ $x=3$

So, S.S = {3, –4}.

(b) Find m, if the roots of the equation $x^2+7x+3m-5=0$, then satisfy the relation $3\alpha-2\beta=4$. **(4)**

Ans $$x^2+7x+3m-5=0$$

Let α, β, be the roots of equation,

$$S=\alpha+\beta=\frac{-b}{a}=\frac{-7}{1}=-7 \qquad \text{(i)}$$

$$P=\alpha\beta=\frac{c}{a}=\frac{3m-5}{1}=3m-5 \qquad \text{(ii)}$$

But we have,

$$3\alpha-2\beta=4$$

From (i)

$$\alpha+\beta=-7$$

$$\beta = -7 - \alpha$$

By putting in the given relation, we get

$$3\alpha - 2(-7 - \alpha) = 4$$

$$3\alpha + 14 + 2\alpha = 4$$

$$5\alpha = 4 - 14$$

$$\alpha = \frac{-10}{5}$$

$$\alpha = -2$$

By putting in (i), we get

$$\alpha + \beta = -7$$

$$-2 + \beta = -7$$

$$\beta = -7 + 2$$

$$\beta = -5$$

By putting the values of α and β in (ii), we get

$$(-2)(-5) = 3m - 5$$

$$10 = 3m - 5$$

$$\Rightarrow \quad 3m = 15$$

$$m = 5$$

Q.6.(a) If a : b = c : d (a, b, c, d ≠ 0), then show that $\frac{a}{b} = \sqrt{\frac{a^2 + c^2}{b^2 + d^2}}$ **(4)**

Ans Given,

$$a : b :: c : d$$

$$\frac{a}{b} = \frac{c}{d}$$

Let, $\frac{a}{b} = \frac{c}{d} = k$

Then,

$$a = b\,k \quad \text{(i)}$$

$$c = d\,k \quad \text{(ii)}$$

Again, Given

$$\frac{a}{b} = \sqrt{\frac{a^2 + c^2}{b^2 + d^2}}$$

$$\text{L.H.S} = \frac{a}{b} \qquad \text{(iii)}$$

By putting (i) in (iii), we get

$$= \frac{bk}{b}$$

$$= k$$

$$\text{R.H.S} = \sqrt{\frac{a^2 + c^2}{b^2 + d^2}} \qquad \text{(iv)}$$

By putting (i) and (ii) in (iv), we get

$$= \sqrt{\frac{(bk)^2 + (dk)^2}{b^2 + d^2}}$$

$$= \sqrt{\frac{b^2k^2 + d^2k^2}{b^2 + d^2}}$$

$$= \sqrt{\frac{k^2(b^2 + d^2)}{(b^2 + d^2)}}$$

$$= \sqrt{k^2}$$

$$= k$$

So,

$$\text{L.H.S} = \text{R.H.S}$$

$$\frac{a}{b} = \sqrt{\frac{a^2 + c^2}{b^2 + d^2}} \quad \text{Proved.}$$

(b) Resolve into partial fractions: $\dfrac{3x - 11}{(x + 3)(x^2 + 1)}$ **(4)**

Ans $$\frac{3x - 11}{(x + 3)(x^2 + 1)} = \frac{Ax + B}{x^2 + 1} + \frac{C}{x + 3}$$

Multiply throughout by $(x^2 + 1)(x + 3)$

$$3x - 11 = (Ax + B)(x + 3) + C(x^2 + 1) \qquad (1)$$

Let $x + 3 = 0 \Rightarrow x = -3$

Put $x = -3$ in (1)

$$3(-3) - 11 = (A(-3) + B)(-3 + 3) + C((-3)^2 + 1)$$

$$-9 - 11 = 0 + C(9 + 1)$$

$$-20 = 10\,C$$

$$C = \frac{-20}{10}$$

$$\boxed{C = -2}$$

Identity can be written as

$$3x - 11 = Ax^2 + 3Ax + Bx + 3B + Cx^2 + C$$

Equating the constants,

$$-11 = 3B + C$$
$$-11 = 3B - 2$$
$$3B = -11 + 2$$
$$3B = -9 \quad \Rightarrow \quad \boxed{B = -3}$$

Equating coefficients of x^2,

$$0 = A + C$$
$$0 = A - 2$$
$$\boxed{A = 2}$$

So, the required partial fraction is:

$$= \frac{2x + (-3)}{x^2 + 1} + \frac{-2}{x + 3}$$

$$= \frac{2x - 3}{x^2 + 1} - \frac{2}{x + 3}$$

Q.7.(a) If U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, A = {1, 3, 5, 7, 9} and B = {2, 3, 5, 7}, then verify $(A \cup B)' = A' \cap B'$. (4)

Ans L.H.S = $(A \cup B)'$

So, $A \cup B = \{1, 3, 5, 7, 9\} \cup \{2, 3, 5, 7\}$

$A \cup B = \{1, 2, 3, 5, 7, 9\}$

$(A \cup B)' = \cup - (A \cup B)$

$= \{1, 2, 3, ..., 10\} - \{1, 2, 3, 5, 7, 9\}$

$= \{4, 6, 8, 10\}$ (i)

Now R.H.S = $A' \cap B'$

$A' = \cup - A$

$A' = \{1, 2, 3, ..., 10\} - \{1, 3, 5, 7, 9\}$

$= \{2, 4, 6, 8, 10\}$

$B' = \cap - B$

$= \{1, 2, 3, ..., 10\} - \{2, 3, 5, 7\}$
$= \{1, 4, 6, 8, 9, 10\}$

Now, $A' \cap B'$
$= \{2, 4, 6, 8, 10\} \cap \{1, 4, 6, 8, 9, 10\}$
$= \{4, 6, 8, 10\}$ (ii) $=$ L.H.S

From (i) & (ii),
Therefore, L.H.S = R.H.S
$\Rightarrow$ $(A \cup B)' = A' \cap B'$

(b) Find the standard deviation "S": **(4)**
9, 3, 8, 8, 9, 8, 9, 18

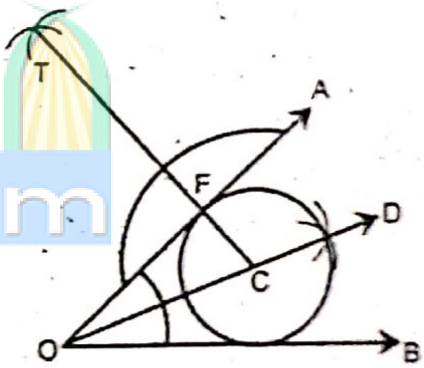
Ans For Answer see Paper 2019 (Group-II), Q.7.(b).

Q.8.(a) Verify the identity: $\dfrac{1+\sin\theta}{1-\sin\theta} - \dfrac{1-\sin\theta}{1+\sin\theta} = 4\tan\theta\sec\theta$. **(4)**

Ans For Answer see Paper 2017 (Group-II), Q.8.(a).

(b) Draw circle which touches both the arms of angle: 45°. **(4)**

Ans



(i) Draw an angle $\angle AOB$ of measure 45°.

(ii) Draw $\overrightarrow{OD}$ a bisector of $\angle A\hat{O}B$ with compass.

(iii) Take any point C on $\overrightarrow{OD}$.

(iv) Draw $\overrightarrow{OT}$ perpendicular to $\overrightarrow{OA}$, intersecting $\overrightarrow{OA}$ at F.

(v) Draw a circle with center C and radius $\overline{CF}$.

(vi) This circle touches both arms of $\angle A\hat{O}B$.

Q.9. A straight line, drawn from the centre of a circle to bisect a chord (which is not a diameter) is perpendicular to the chord. **(8)**

Ans **Given:**

M is the mid-point of any chord $\overline{AB}$ of a circle with centre at O.

Where chord $\overline{AB}$ is not the diameter of the circle.

To Prove:

$\overline{OM} \perp$ the chord $\overline{AB}$.

Construction:

Join A and B with centre O.

Write $\angle 1$ and $\angle 2$ as shown in the figure.

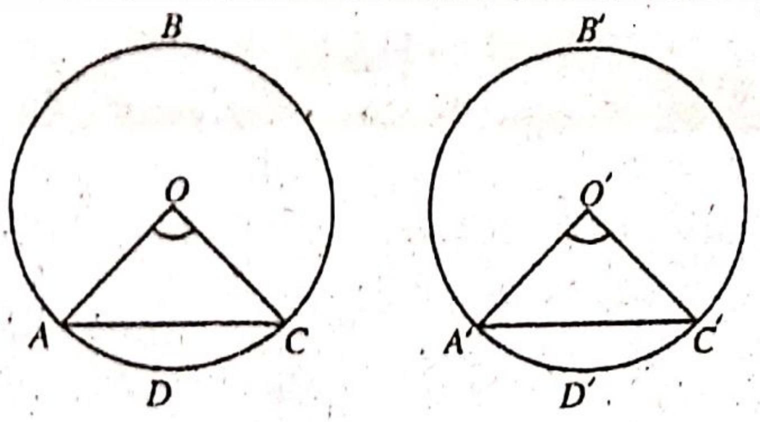
Proof:

Statements	Reasons
In $\Delta OAM \leftrightarrow \Delta OBM$	
$m\overline{OA} = m\overline{OB}$	Radii of the same circle
$m\overline{AM} = m\overline{BM}$	Given
$m\overline{OM} = m\overline{OM}$	Common
$\therefore$ $\Delta OAM \cong \Delta OBM$	S.S.S $\cong$ S.S.S
$\Rightarrow$ $m\angle 1 = M\angle 2$ (i)	Corresponding sides of congruent triangles.
i.e., $m\angle 1 + m\angle 2 = m\angle AMB = 180^\circ$ (ii)	Adjacent supplementary angles
$\therefore$ $m\angle 1 = m\angle 2 = 90^\circ$	From (i) and (ii)
i.e., $\overline{OM} \perp \overline{AB}$	

OR

If the angles subtended by two chords of a circle (or congruent circles) at the centre (corresponding centres) are equal, the chords are equal.

Ans :



Given:

ABCD and A′B′C′D′ are two congruent circles with centres.

O and O′ respectively. $\overline{AC}$ and $\overline{A'C'}$ are chords of circles ABCD and A′B′C′D′, respectively and $m\angle AOC = m\angle A'O'C'$.

To Prove:

$$m\overline{AC} = m\overline{A'C'}$$

Proof:

Statements	Reasons
In $\Delta OAC \leftrightarrow \Delta O'A'C'$	
$m\overline{OA} = m\overline{O'A'}$	Radii of the congruent circles
$m\angle AOC = m\angle A'O'C'$	Given
$m\overline{OC} = m\overline{O'C'}$	Radii of congruent circles
$\therefore$ $\Delta OAC \cong \Delta O'A'C'$	SAS $\cong$ SAS
Hence $m\overline{AC} = m\overline{A'C'}$	